Beyond Attenuation

Asymptotic Rejection of Modelled Disturbances in Uncertain Linear Plants

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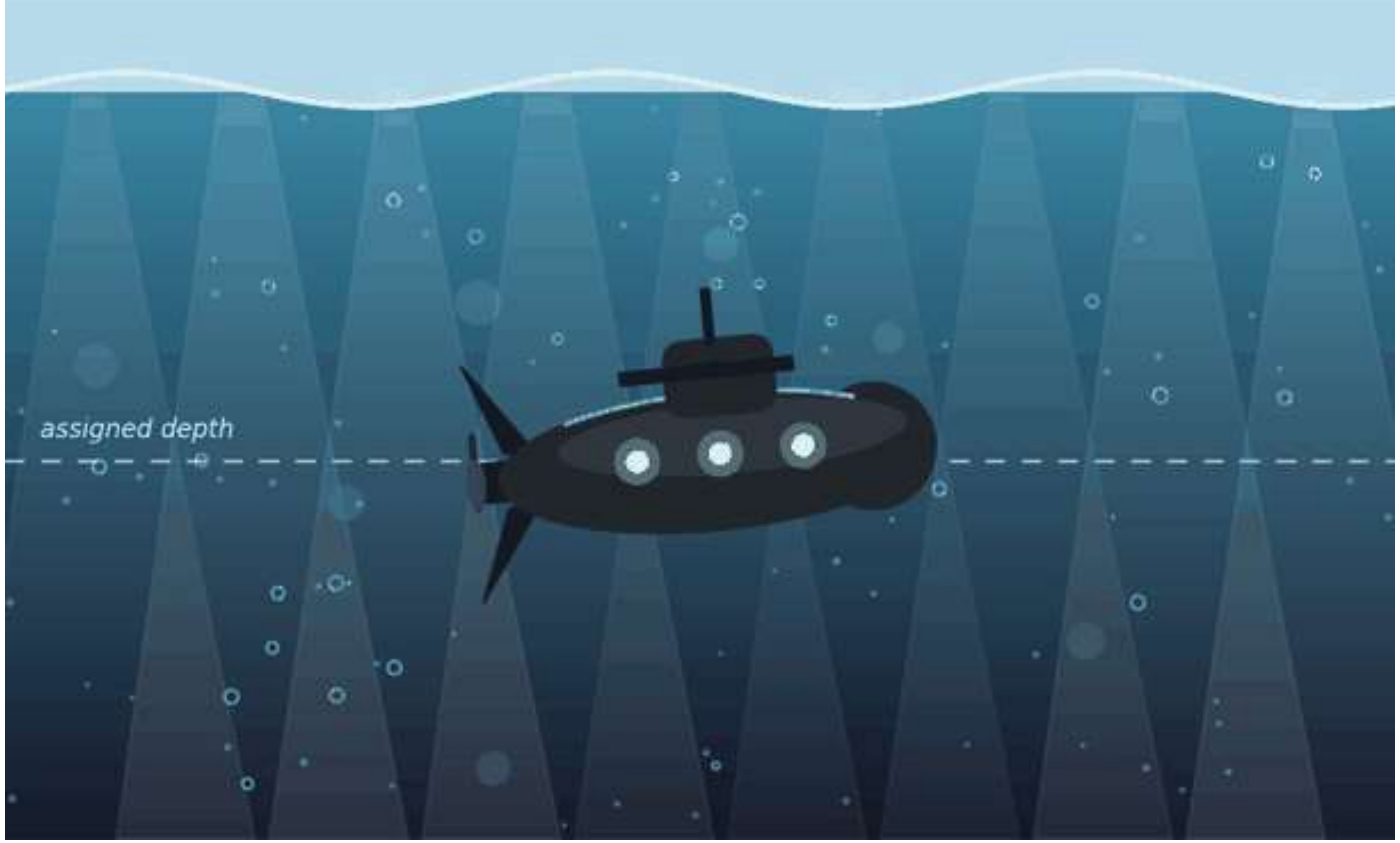
Why disturbance rejection?

A vehicle must hold a target depth.

A conventional controller only **attenuates** the disturbance so the vehicle never quite settles on the target.

This talk: how to **reject** modelled disturbances exactly when its model is not fully known.



Outline

1. **Existing works**
   - ✔ Conventional disturbance observers
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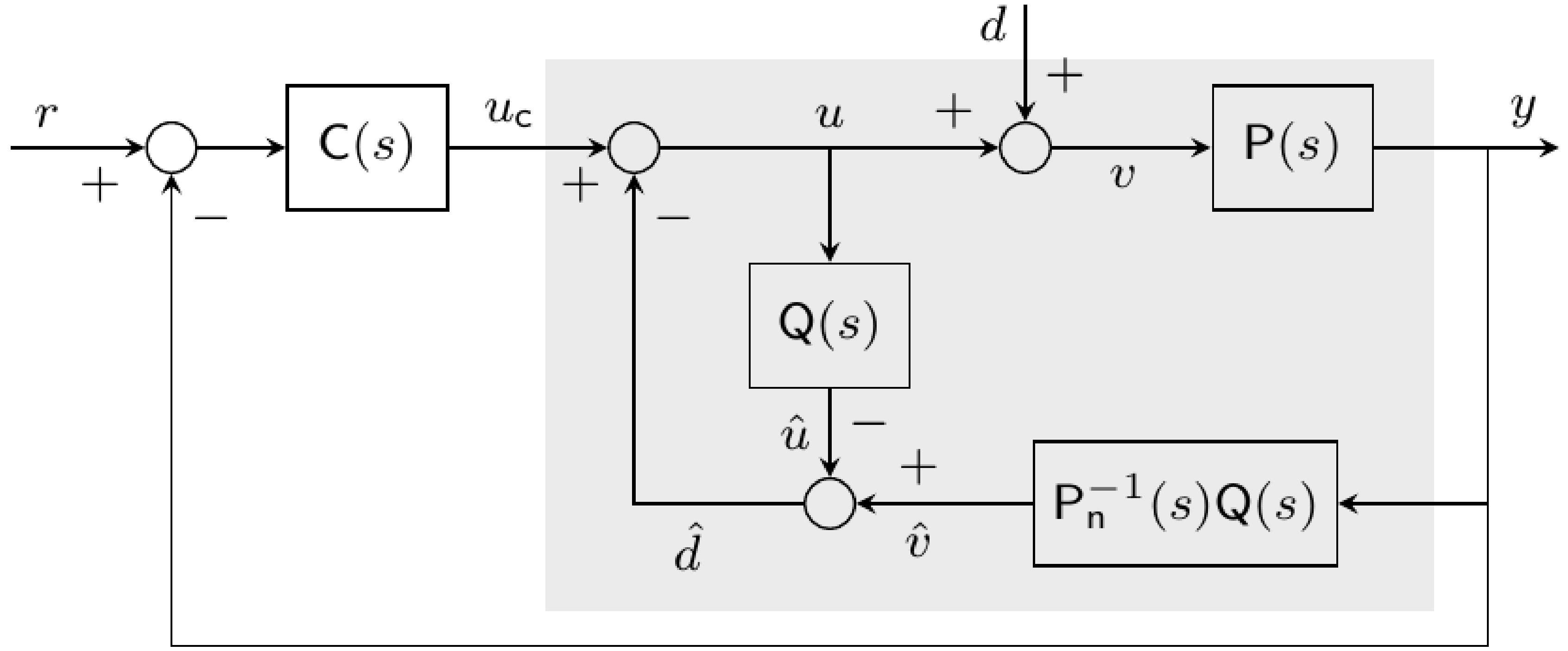
Conventional disturbance observers (DOBs)

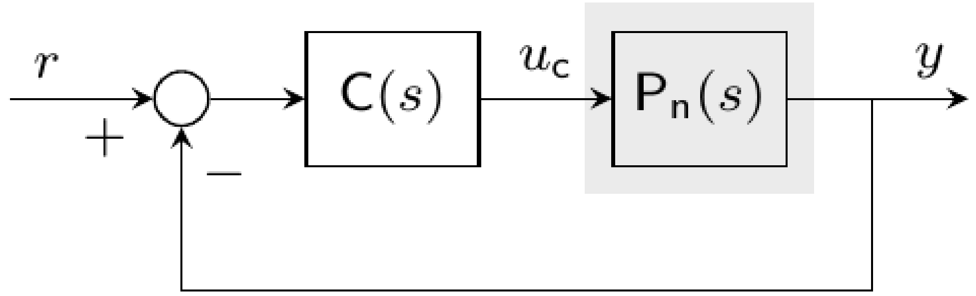
Existing Works



- P: Uncertain plant.
- P_n: Nominal plant.
- C: Outer-loop controller designed for P_n.
- Q: Low-pass filter with unity DC gain.
- d: external (unmodeled and bounded) disturbance.

- **Closed-loop system:**

$$y = \frac{PP_nC}{(1-Q)P_n + P(Q+P_nC)} r + \frac{(1-Q)PP_n}{(1-Q)P_n + P(Q+P_nC)} d.$$



- **In low frequency range:** $Q(jw) \approx 1$ and

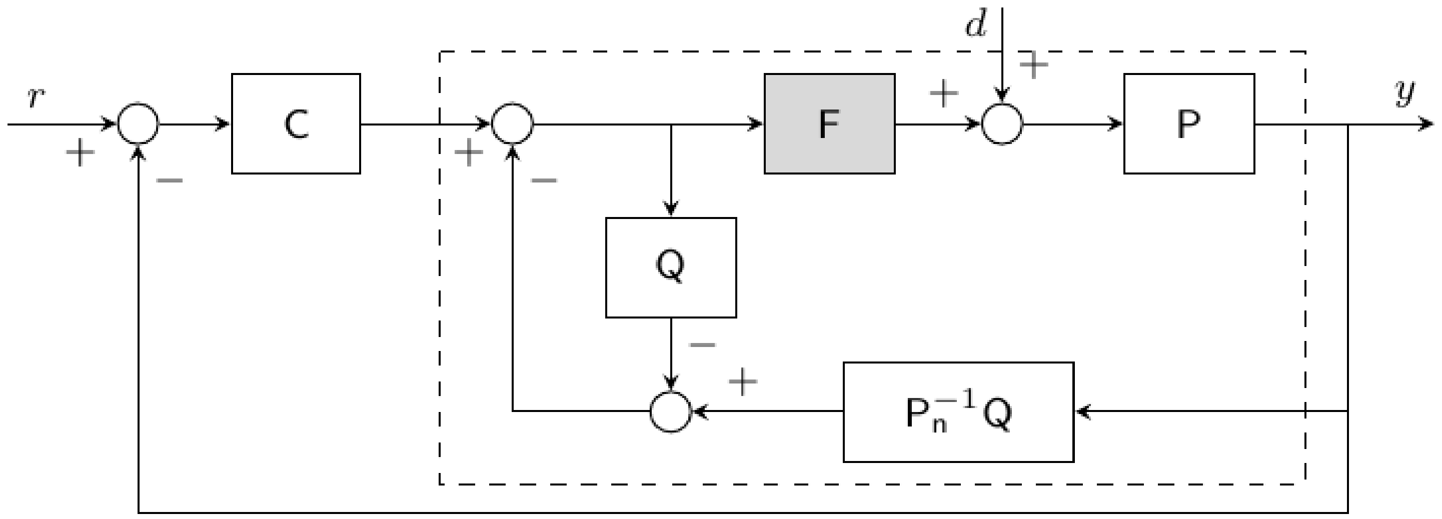
$$y(jw) \approx \frac{P_n(jw)C(jw)}{1+P_n(jw)C(jw)} r(jw).$$

Internal-model-based DOBs: Modeled disturbances

Existing Works

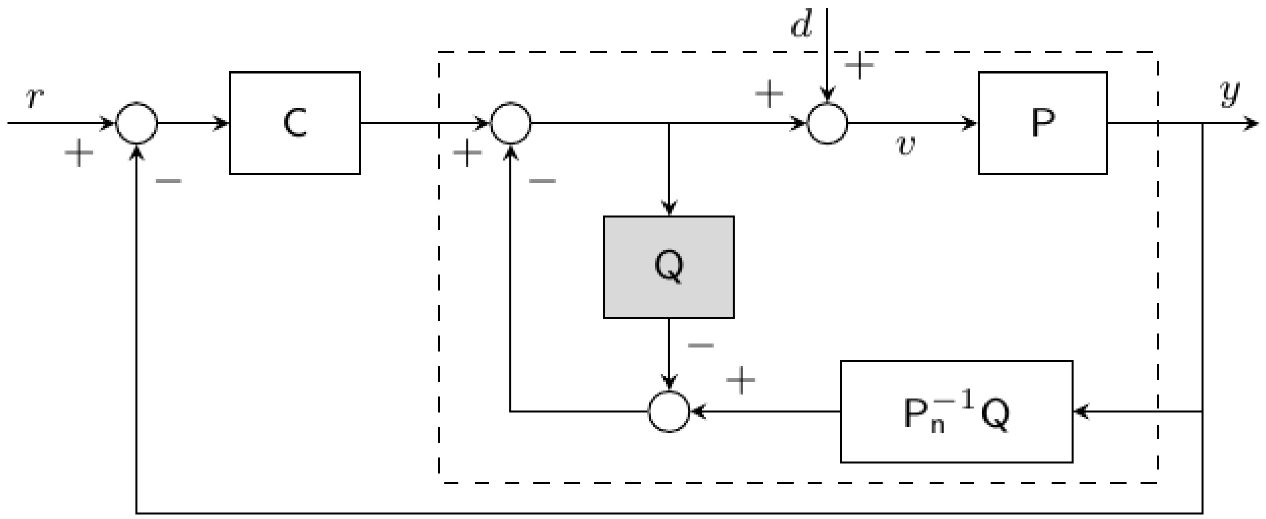
Adding an additional filter in feed-through path: [1]

- Disturbance under consideration:
$$\dot{w} = Sw, \qquad d = [1 \;\; 0 \;\; \cdots \;\; 0]w.$$
- Increases the order and structural complexity of the DOB.



Embedding disturbance model in Q-filter: [2,3,4]

- Disturbance under consideration:
$$d(t) = \sum_{i=0}^{k_t} M_i t^i + \sum_{j=1}^{k_s} N_i \sin(\sigma_j t + \phi_j).$$
- Constraint: $\sigma_i \neq \sigma_j$.
Otherwise, the DOB design is infeasible.
- Coefficients of the numerator of Q(s)-filter depend on τ *implicitly*.



[1] Kim, Park, & Shim, "Robust disturbance rejection for uncertain linear system with TV exosystem: A DOB approach," *AUT*, 2025.
[2] Joo, Park, Back, & Shim, "Embedding internal model in DOB with robust stability," *TAC*, 2016.
[3] Park, Shim, & Joo, "Recovering nominal tracking performance in an asymptotic sense for uncertain linear systems," *SIAM*, 2018.
[4] Kim, Joo, & Park, "Design of HGO-based High-order internal model DOB," Proc. of *ICCAS*, 2024.

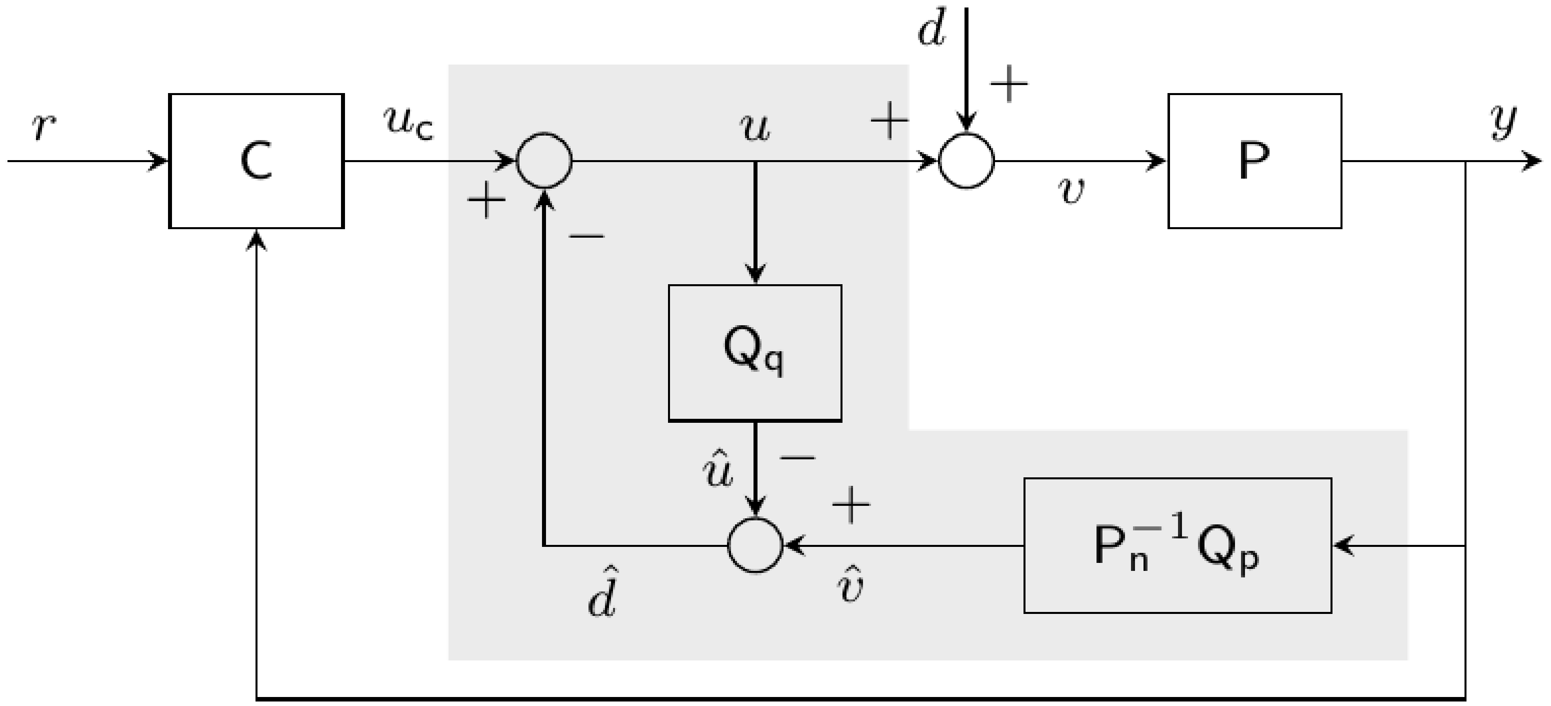
Problem formulation: Plant and modeled disturbance

Main Result

Plant P in Byrnes-Isidori's normal form:

$$\dot{x} = A_\nu x + B_\nu \left(\phi^\top x + g(u+d)\right),$$
$$y = C_\nu x,$$

where $x \in \mathbb{R}^\nu$ and $u, y, d \in \mathbb{R}$.



Assumption (Plant)

- *Uncertain* $\phi \in \mathbb{R}^\nu$ *lies in a known compact set.*
- $g \in [g_{\min}, g_{\max}]$, *where* $g_{\min} > 0$ *and* $g_{\max}$ *are known.*

$$A_\nu := \begin{bmatrix} 0_{\nu-1} & I_{\nu-1} \\ 0 & 0_{\nu-1}^\top \end{bmatrix}, \quad B_\nu := \begin{bmatrix} 0_{\nu-1} \\ 1 \end{bmatrix}, \quad C_\nu := \begin{bmatrix} 1 & 0_{\nu-1}^\top \end{bmatrix}.$$

Modeled disturbance:

$$\begin{aligned} \dot{w} &= Sw, \\ d &= Rw, \end{aligned} \qquad w \in \mathbb{R}^m.$$

Assumption (Disturbance)

- S *and* R *are unknown but* $\operatorname{Re}\lambda_i(S) \geq 0$ *for* $i = 1, \ldots, m$.
- *The minimal polynomial of* S *is known, where*

$$\Delta_{\mathrm{w}}(s) := s^\mu + a_{\mu-1}s^{\mu-1} + \cdots + a_1 s + a_0.$$

Problem formulation: Nominal systems and the problem

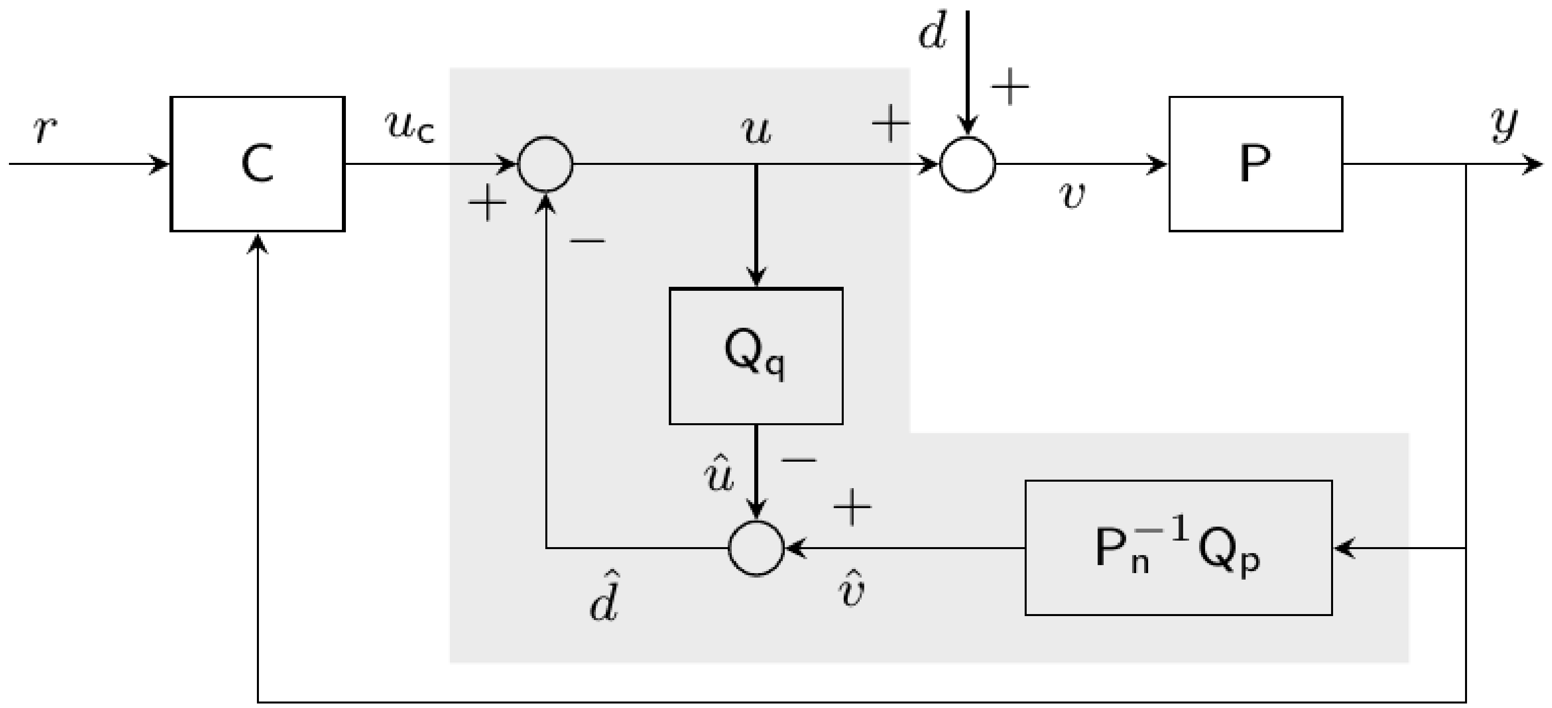
Main Result

Nominal counterpart $\mathsf{P_n}$ of P:

$$\dot{x}_{\mathsf{n}} = A_\nu x_{\mathsf{n}} + B_\nu \left(\phi_{\mathsf{n}}^\top x_{\mathsf{n}} + g_{\mathsf{n}} u_{\mathsf{n}}\right),$$
$$y_{\mathsf{n}} = C_\nu x_{\mathsf{n}}.$$

Note: Nominal plant contains no uncertainty and disturbance.

Nominal controller: $\dot{\zeta}_{\mathsf{n}} = F\zeta_{\mathsf{n}} + G_1 y_{\mathsf{n}} + G_2 r,$
$u_{\mathsf{n}} = J\zeta_{\mathsf{n}} + K_1 y_{\mathsf{n}} + K_2 r.$



Assumption (Nominal closed-loop system)

The nominal closed-loop system with $\chi_{\mathsf{n}} := [x_{\mathsf{n}}; \zeta_{\mathsf{n}}]$ is asymptotically stable under $r \equiv 0$, i.e., $\mathcal{A}_s$ is Hurwitz.

$$\dot{\chi}_{\mathsf{n}} = \mathcal{A}_s \chi_{\mathsf{n}} + \mathcal{B}_s r := \begin{bmatrix} A_\nu + B_\nu\left(\phi_{\mathsf{n}}^\top + g_{\mathsf{n}} K_1 C_\nu\right) & g_{\mathsf{n}} B_\nu J \\ G_1 C_\nu & F \end{bmatrix} \chi_{\mathsf{n}} + \begin{bmatrix} g_{\mathsf{n}} B_\nu K_2 \\ G_2 \end{bmatrix} r.$$

DOB problem

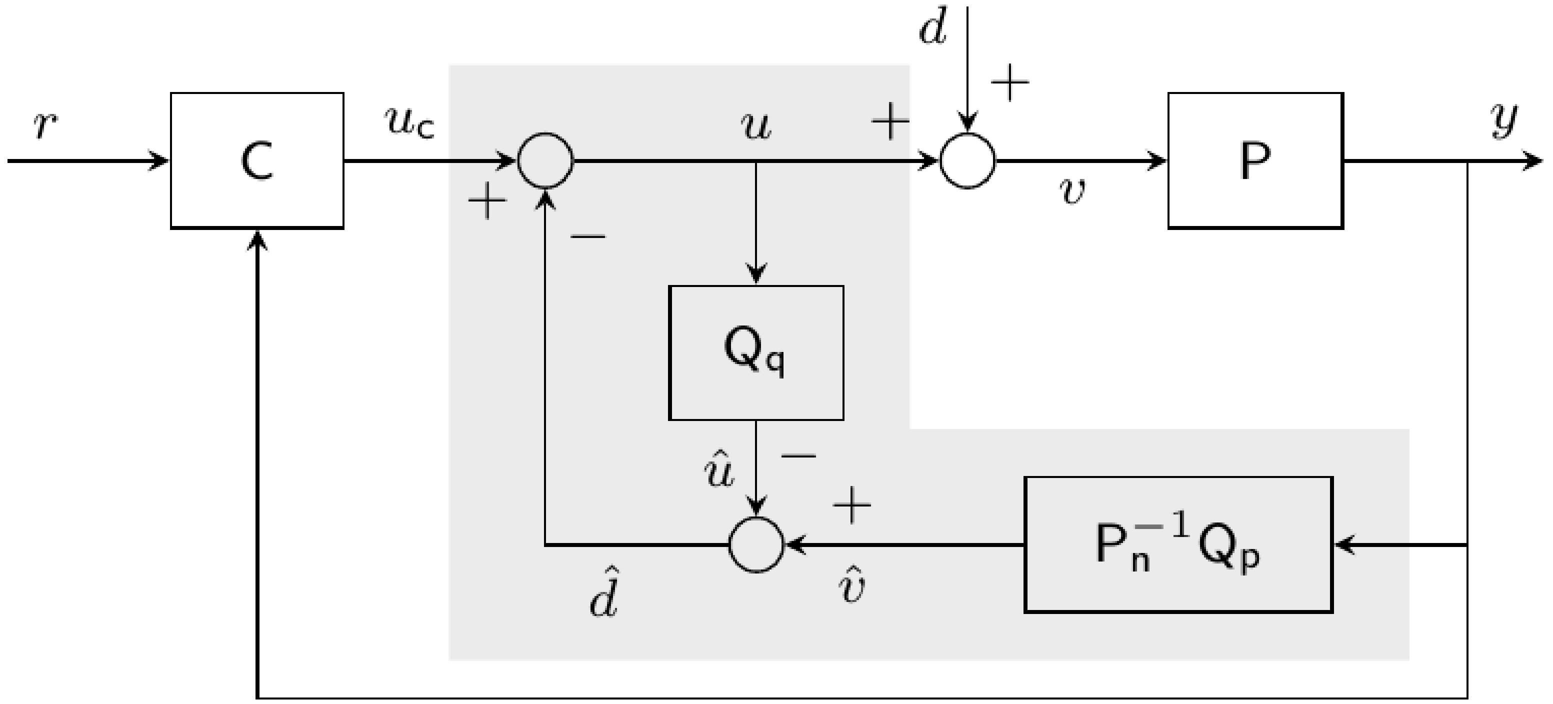
Design a DOB ($\mathsf{Q_q}$ and $\mathsf{P_n^{-1}Q_p}$) s.t. the actual closed-loop system is asymptotically stable under $r \equiv 0$.

Internal-model-based design: $\mathsf{P}_\mathsf{n}^{-1}\mathsf{Q}_\mathsf{p}$

Main Result

Nominal plant P_n:

$$\dot{x}_\mathsf{n} = A_\nu x_\mathsf{n} + B_\nu\left(\phi_\mathsf{n}^\top x_\mathsf{n} + g_\mathsf{n} u_\mathsf{n}\right) \qquad y_\mathsf{n} = C_\nu x_\mathsf{n} \quad \Longrightarrow \quad \begin{aligned} y_\mathsf{n} &= x_{\mathsf{n},1} \\ \dot{x}_{\mathsf{n},i} &= x_{\mathsf{n},i+1} \\ \dot{x}_{\mathsf{n},\nu} &= \phi_\mathsf{n}^\top x_\mathsf{n} + g_\mathsf{n} u_\mathsf{n} \end{aligned}$$



Nominal inverse $\mathsf{P}_\mathsf{n}^{-1}$:

$$u_\mathsf{n} = \frac{1}{g_\mathsf{n}}\left(y_\mathsf{n}^{(\nu)} - \phi_\mathsf{n}^\top[y_\mathsf{n}; \dot{y}_\mathsf{n}; \cdots; y_\mathsf{n}^{(\nu-1)}]\right).$$

Implementation of $\mathsf{P}_\mathsf{n}^{-1}\mathsf{Q}_\mathsf{p}$:

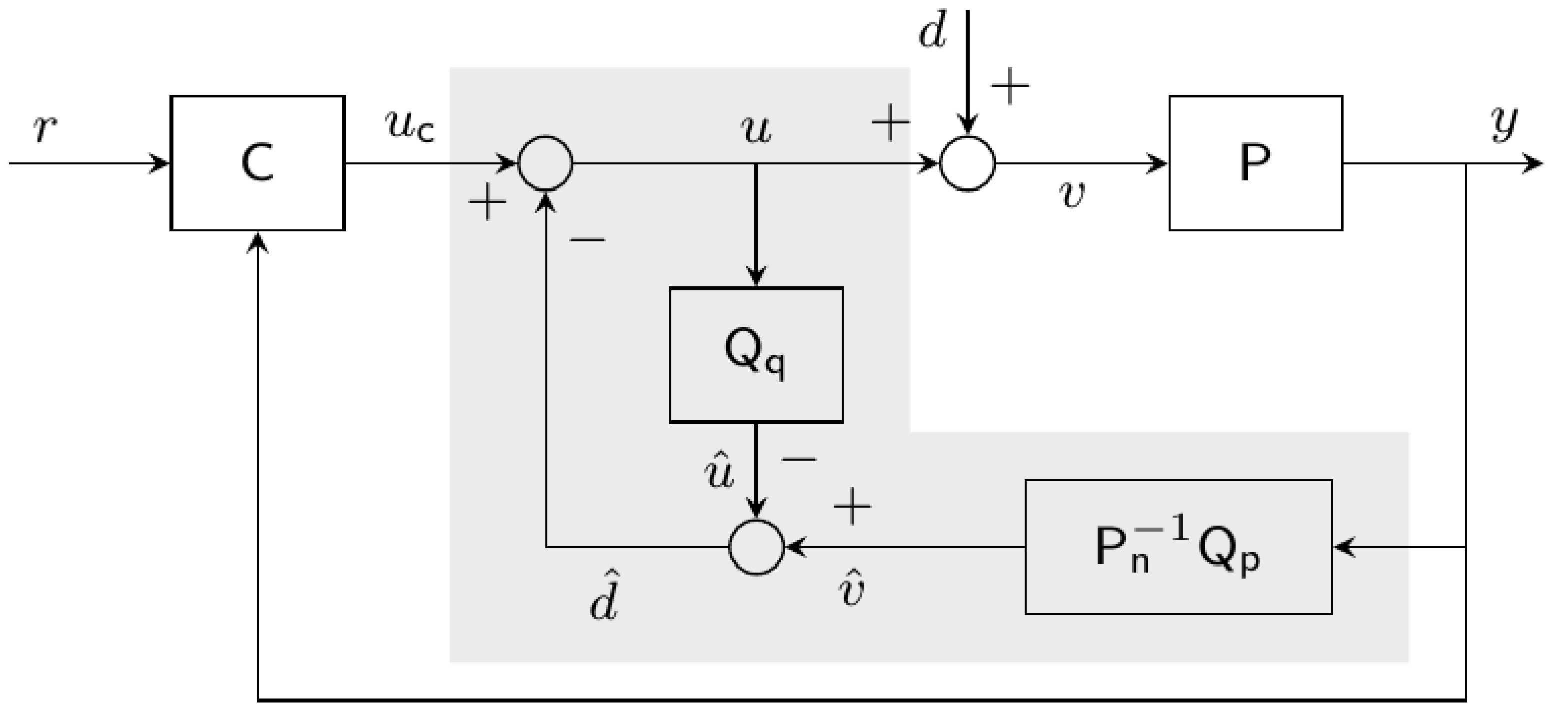
$$\begin{aligned} \dot{p} &= A_\nu p + H(y - C_\nu p), \\ \hat{v} &= \frac{1}{g_\mathsf{n}}\left(\dot{p}_\nu - \phi_\mathsf{n}^\top p\right), \end{aligned} \qquad H := \left[\frac{\beta_{\nu-1}}{\tau}; \frac{\beta_{\nu-2}}{\tau^2}; \cdots; \frac{\beta_0}{\tau^\nu}\right], \quad A_\nu = \begin{bmatrix} 0_{\nu-1} & I_{\nu-1} \\ 0 & 0_{\nu-1}^\top \end{bmatrix}, \; C_\nu = \begin{bmatrix} 1 & 0_{\nu-1}^\top \end{bmatrix},$$

where $p := [p_1; \cdots; p_\nu] \in \mathbb{R}^\nu$, $\hat{v} \in \mathbb{R}$, and $\Delta_\mathsf{p}(s) = s^\nu + \beta_{\nu-1}s^{\nu-1} + \cdots + \beta_1 s + \beta_0$ is a Hurwitz polynomial.

Internal-model-based design: $\mathsf{Q_q}$ and C

Main Result

Nominal controller:
$$\dot\zeta_{\mathsf{n}} = F\zeta_{\mathsf{n}} + G_1 y_{\mathsf{n}} + G_2 r,$$
$$u_{\mathsf{n}} = J\zeta_{\mathsf{n}} + K_1 y_{\mathsf{n}} + K_2 r.$$



Outer-loop controller C**:**

$$\dot\zeta = F\zeta + G_1 y + G_2 r,$$
$$u_{\mathsf{c}} = J\zeta + K_1 y + K_2 r.$$

Design of $\mathsf{Q_q}$**:** With $q := [q_1; \cdots ; q_\mu] \in \mathbb{R}^\mu$,

$$\dot q = A_{\mathsf{q}} q + B_{\mathsf{q}} u, \quad \hat u = C_\mu q, \quad u = u_{\mathsf{c}} - (\hat v - \hat u),$$

$$A_{\mathsf{q}} := \begin{bmatrix} -\frac{\alpha_{\mu-1}}{\tau} & 1 & 0 & \cdots & 0 \\ -\frac{\alpha_{\mu-2}}{\tau^2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{\alpha_1}{\tau^{\mu-1}} & 0 & 0 & \cdots & 1 \\ -\frac{\alpha_0}{\tau^\mu} & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_{\mathsf{q}} := \begin{bmatrix} \frac{\alpha_{\mu-1}}{\tau} - a_{\mu-1} \\ \frac{\alpha_{\mu-2}}{\tau^2} - a_{\mu-2} \\ \vdots \\ \frac{\alpha_1}{\tau^{\mu-1}} - a_1 \\ \frac{\alpha_0}{\tau^\mu} - a_0 \end{bmatrix}, \quad C_\mu := \begin{bmatrix} 1 & 0_{\mu-1}^\top \end{bmatrix},$$

where $\Delta_{\mathsf{q}}(s) = s^\mu + \alpha_{\mu-1}s^{\mu-1} + \cdots + \alpha_1 s + \alpha_0$ is a Hurwitz polynomial and
$\Delta_{\mathsf{w}}(s) = s^\mu + a_{\mu-1}s^{\mu-1} + \cdots + a_1 s + a_0$ is the minimal polynomial of S.

Stability analysis: Internal model principle

Main Result

Recall that $\dot{w} = Sw$, $d = Rw$, and $\Delta_{\mathrm{w}}(s) = s^{\mu} + a_{\mu-1}s^{\mu-1} + \cdots + a_1 s + a_0$.

Define $\Pi_1 := -R$, $\Pi_i := \Pi_{i-1}S + a_{\mu+1-i}\Pi_1$ for $i = 2, \ldots, \mu$, and $\Pi := [\Pi_1; \cdots; \Pi_\mu] \in \mathbb{R}^{\mu \times m}$.

Then, since $\Delta_{\mathrm{w}}(S) = 0$ and $\Pi_{i-1}S = -a_{\mu+1-i}\Pi_1 + \Pi_i$,

$$\begin{aligned}
\Pi_1 S &= -a_{\mu-1}\Pi_1 + \Pi_2, \\
&\vdots \\
\Pi_{\mu-1} S &= -a_1\Pi_1 + \Pi_\mu, \\
\Pi_\mu S + a_0\Pi_1 &= \Pi_{\mu-1}S^2 + a_1\Pi_1 S + a_0\Pi_1 = \cdots \\
&= \Pi_1\left(S^{\mu} + a_{\mu-1}S^{\mu-1} + \cdots + a_1 S + a_0 I\right) = 0,
\end{aligned}$$

$$\begin{aligned}
\Pi_1 S &= -a_{\mu-1}\Pi_1 + \Pi_2, \\
&\vdots \\
\Pi_{\mu-1} S &= -a_1\Pi_1 + \Pi_\mu, \\
\Pi_\mu S &= -a_0\Pi_1.
\end{aligned}$$

Matrix identity (internal model principle):

$$\begin{bmatrix} \Pi_1 \\ \vdots \\ \Pi_{\mu-1} \\ \Pi_\mu \end{bmatrix} S = \begin{bmatrix} -a_{\mu-1} & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & 0 & 0 & \cdots & 1 \\ -a_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \Pi_1 \\ \vdots \\ \Pi_{\mu-1} \\ \Pi_\mu \end{bmatrix} \iff \begin{aligned} \Pi S &= (A_\mu - aC_\mu)\,\Pi, \\ -R &= C_\mu \Pi \quad (= [1 \quad 0 \quad \cdots \quad 0]\,\Pi), \end{aligned}$$

where $a := [a_{\mu-1}; \cdots; a_1; a_0] \in \mathbb{R}^{\mu}$ and $A_\mu := [0_{\mu-1} \;\; I_{\mu-1};\; 0 \;\; 0_{\mu-1}^{\top}]$.

Stability analysis: Standard form of singular perturbations

Main Result

Closed-loop system:

$$\dot{x} = A_\nu x + B_\nu(\phi^\top x + g(u+d)), \qquad y = C_\nu x,$$
$$\dot{\zeta} = F\zeta + G_1 y + G_2 r, \qquad u_{\mathsf{c}} = J\zeta + K_1 y + K_2 r,$$
$$\dot{p} = A_\nu p + H(y - C_\nu p), \qquad \hat{v} = \frac{1}{g_{\mathsf{n}}}\left(\dot{p}_\nu - \phi_{\mathsf{n}}^\top p\right),$$
$$\dot{q} = A_{\mathsf{q}} q + B_{\mathsf{q}} u, \qquad u = u_{\mathsf{c}} + \hat{u} - \hat{v}, \qquad \hat{u} = C_\mu q.$$

Coordinate changes: $\hat{q} := \frac{1}{\tau}\mathcal{T}(q - \Pi w)$ and $\hat{p} := \Upsilon^{-1}(x - p)$,
where $\mathcal{T} := \mathrm{diag}\{\tau, \tau^2, \ldots, \tau^\mu\}$ and $\Upsilon := \mathrm{diag}\{\tau^\nu, \tau^{\nu-1}, \ldots, \tau\}$.

Standard form:

$$\dot{x} = \left(A_\nu + B_\nu\phi^\top\right)x - \frac{g}{g_{\mathsf{n}}}\beta_0 B_\nu C_\nu \hat{p} + gB_\nu C_\mu \hat{q} + gB_\nu f(x,\zeta,r,\hat{p},\tau),$$
$$\dot{\zeta} = F\zeta + G_1 C_\nu x + G_2 r,$$
$$\tau\begin{bmatrix}\dot{\hat{p}}\\ \dot{\hat{q}}\end{bmatrix} = \mathcal{A}_f\begin{bmatrix}\hat{p}\\ \hat{q}\end{bmatrix} + \begin{bmatrix} B_\nu\phi^\top x + gB_\nu f(x,\zeta,r,\hat{p},\tau) \\ \alpha f(x,\zeta,r,\hat{p},\tau) - \mathcal{T}a\left(C_\mu\hat{q} - \frac{\beta_0}{g_{\mathsf{n}}}C_\nu\hat{p} + f(x,\zeta,r,\hat{p},\tau)\right)\end{bmatrix},$$

where $f(\cdot) := J\zeta + K_1 C_\nu x + K_2 r + \frac{1}{g_{\mathsf{n}}}\phi_{\mathsf{n}}^\top(x - \Upsilon\hat{p})$, $\alpha := [\alpha_{\mu-1};\cdots;\alpha_1;\alpha_0]$, $\beta := [\beta_{\nu-1};\cdots;\beta_1;\beta_0]$, and

$$\mathcal{A}_f := \begin{bmatrix} A_\nu - \beta C_\nu - \frac{g}{g_{\mathsf{n}}}\beta_0 B_\nu C_\nu & gB_\nu C_\mu \\ -\frac{\beta_0}{g_{\mathsf{n}}}\alpha C_\nu & A_\mu \end{bmatrix}.$$

Note: No disturbance signal w appears in the above equation.

Stability analysis: Quasi-steady-states

Main Result

Standard form:

$$\begin{aligned}
\dot{x} &= \left(A_\nu + B_\nu \phi^\top\right) x - \frac{g}{g_{\mathsf{n}}}\beta_0 B_\nu C_\nu \hat{p} + g B_\nu C_\mu \hat{q} + g B_\nu f(x,\zeta,r,\hat{p},\tau),\\
\dot{\zeta} &= F\zeta + G_1 C_\nu x + G_2 r,\\
\tau \begin{bmatrix}\dot{\hat{p}}\\ \dot{\hat{q}}\end{bmatrix} &= \mathcal{A}_f \begin{bmatrix}\hat{p}\\ \hat{q}\end{bmatrix} + \begin{bmatrix} B_\nu \phi^\top x + g B_\nu f(x,\zeta,r,\hat{p},\tau)\\ \alpha f(x,\zeta,r,\hat{p},\tau) - \mathcal{T} a \left(C_\mu \hat{q} - \frac{\beta_0}{g_{\mathsf{n}}} C_\nu \hat{p} + f(x,\zeta,r,\hat{p},\tau)\right)\end{bmatrix}.
\end{aligned}$$

Quasi-steady-states: By letting $\tau = 0$,

$$\begin{aligned}
\hat{p}_i^\star &= \frac{\beta_{\nu-i+1}}{\beta_0} g_{\mathsf{n}} \left(J\zeta + K_1 C_\nu x + K_2 r + \frac{1}{g_{\mathsf{n}}}\phi_{\mathsf{n}}^\top x\right), && i = 1,\ldots,\nu, \quad \beta_\nu := 1,\\
\hat{q}_1^\star &= \frac{1}{g}\left(\phi_{\mathsf{n}} - \phi\right)^\top x + \frac{g_{\mathsf{n}}}{g}\left(J\zeta + K_1 C_\nu x + K_2 r\right),\\
\hat{q}_j^\star &= 0, && j = 2,\ldots,\mu.
\end{aligned}$$

Note: Quasi-steady-states do not depend on the signal w!

Stability analysis: Reduced and boundary layer systems

Main Result

Reduced system: With $\chi := [x; \zeta]$,

$$\dot{\chi} = \begin{bmatrix} A_\nu + B_\nu \left(\phi_{\mathsf{n}}^\top + g_{\mathsf{n}} K_1 C_\nu\right) & g_{\mathsf{n}} B_\nu J \\ G_1 C_\nu & F \end{bmatrix} \chi + \begin{bmatrix} g_{\mathsf{n}} B_\nu K_2 \\ G_2 \end{bmatrix} r = \mathcal{A}_s \chi + \mathcal{B}_s r.$$

Nominal closed-loop system: $\dot{\chi}_{\mathsf{n}} = \mathcal{A}_s \chi_{\mathsf{n}} + \mathcal{B}_s r$.

Boundary layer system: By letting $\hat{t} := \frac{t}{\tau}$ and $\tau = 0$,

$$\begin{bmatrix} \hat{p}' \\ \hat{q}' \end{bmatrix} = \begin{bmatrix} A_\nu - \beta C_\nu - \frac{g}{g_{\mathsf{n}}} \beta_0 B_\nu C_\nu & g B_\nu C_\mu \\ -\frac{\beta_0}{g_{\mathsf{n}}} \alpha C_\nu & A_\mu \end{bmatrix} \begin{bmatrix} \hat{p} \\ \hat{q} \end{bmatrix} = \mathcal{A}_f \begin{bmatrix} \hat{p} \\ \hat{q} \end{bmatrix}.$$

Lemma (Stability of boundary layer system)

The characteristic polynomial of $\mathcal{A}_f$ is given by

$$\begin{aligned} \Delta_f(s) &= s^{\nu+\mu} + \beta_{\nu-1} s^{\nu+\mu-1} + \cdots + \beta_1 s^{\mu+1} + \beta_0 s^\mu + \frac{g}{g_{\mathsf{n}}} \beta_0 \left(s^\mu + \alpha_{\mu-1} s^{\mu-1} + \cdots + \alpha_1 s + \alpha_0\right) \\ &= s^\mu \Delta_{\mathsf{p}}(s) + \frac{g}{g_{\mathsf{n}}} \beta_0 \Delta_{\mathsf{q}}(s). \end{aligned}$$

- $\Delta_f(s)$ can always be made Hurwitz for all $g \in [g_{\min}, g_{\max}]$ by suitably choosing α and β.

Stability analysis: Main theorem

Main Result

Theorem (Main theorem)

Suppose that

- *the nominal closed-loop system is asymptotically stable (stability of reduced system);*
- $\Delta_{\mathsf{p}}(s)$ *and* $\Delta_{\mathsf{q}}(s)$ *are Hurwitz polynomial; and*
- $\Delta_f(s)$ *is a Hurwitz polynomial (stability of boundary layer system).*

Then under $r \equiv 0$, $\exists \tau^\star > 0$ *such that for all* $\tau \in (0, \tau^\star)$,

- *the actual closed-loop system is asymptotically stable; and*
- *the following hold.*

$$\lim_{t\to\infty} \left\| \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \right\| = 0, \qquad \lim_{t\to\infty} \|q(t) - \Pi w(t)\| = 0.$$

- Embedding of an internal model of disturbance into the $\mathsf{Q_q}$-filter is confirmed by the asymptotic behavior.
- Since $u + d = u_{\mathsf{c}} + \hat{u} - \hat{v} + d$, $\hat{u} = C_\mu q$, $d = Rw$, and $-R = C_\mu \Pi$, we have

$$u(t) + d(t) = u_{\mathsf{c}}(t) + C_\mu q(t) - \hat{v}(t) + d(t) \longrightarrow u_{\mathsf{c}}(t) + C_\mu \Pi w(t) - \hat{v}(t) + d(t) = u_{\mathsf{c}}(t) - \hat{v}(t).$$

- It is not necessary for the $\mathsf{Q_q}$-dynamics to contain a zero eigenvalue.

Mechanical positioning system

Computer Simulation

Plant P **[5,6]:**

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -\mathfrak{B}/\mathfrak{J} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/\mathfrak{J} \end{bmatrix} (u + d), \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x,$$

where $\mathfrak{J} \in [0.2, 0.4]$ $\mathrm{V/(m/s^2)}$ and $\mathfrak{B} \in [0.1, 0.2]$ $\mathrm{V/(m/s)}$.

Disturbance model:

$$\begin{aligned} \dot{w} &= Sw, \\ d &= Rw, \end{aligned} \quad S := \begin{bmatrix} S_\sigma & I_2 \\ 0 & S_\sigma \end{bmatrix}, \quad S_\sigma := \begin{bmatrix} 0 & \sigma \\ -\sigma & 0 \end{bmatrix},$$

where $R \in [-1, 1]^2$ and $\sigma = 30$ rad/s.

Nominal plant $\mathsf{P_n}$**:** $\mathfrak{J}_\mathsf{n} = 0.3$ $\mathrm{V/(m/s^2)}$ and $\mathfrak{B}_\mathsf{n} = 0.15$ $\mathrm{V/(m/s)}$.

Nominal controller: With $r(t) = 0.03$ m for $t \geq 0$,

$$\dot{\zeta}_\mathsf{n} = -100.5\zeta_\mathsf{n} - 5525 y_\mathsf{n} + 500r, \qquad u_\mathsf{n} = -15\zeta_\mathsf{n} - 900 y_\mathsf{n} + 150r.$$

$\mathsf{Q_p}$**-filter:** $\Delta_\mathsf{p}(s) = s^2 + 2s + 1$.

$\mathsf{Q_q}$**-filter:** $\Delta_\mathsf{q}(s) = s^4 + 2s^3 + 0.2s^2 + 0.04s + 0.0005$.

$\Longrightarrow \Delta_f(s)$ is Hurwitz for all $g \in [2.5, 5]$.

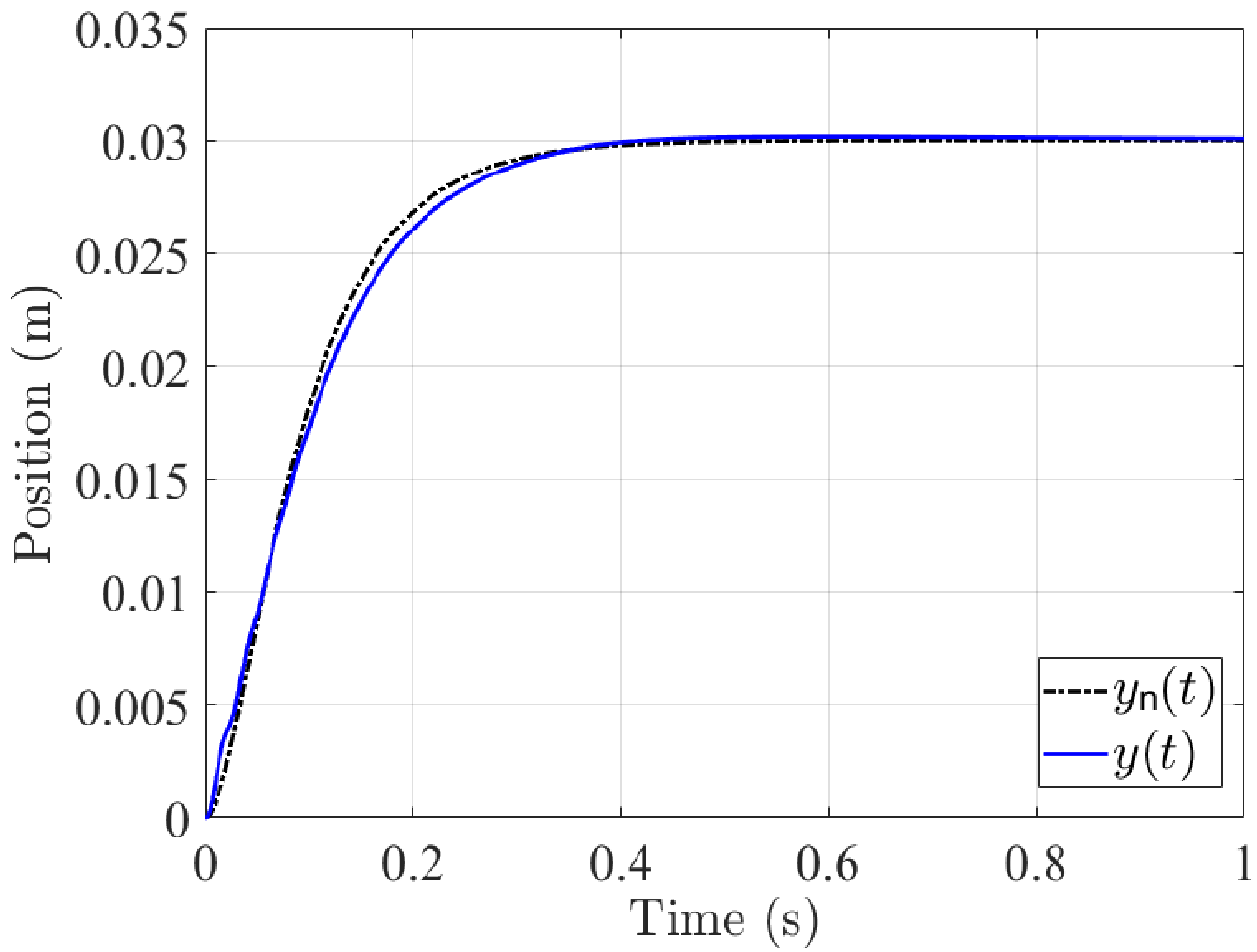
[5] Kim & Chung, "Advanced disturbance observer design for mechanical positioning systems," *TIE*, 2003.

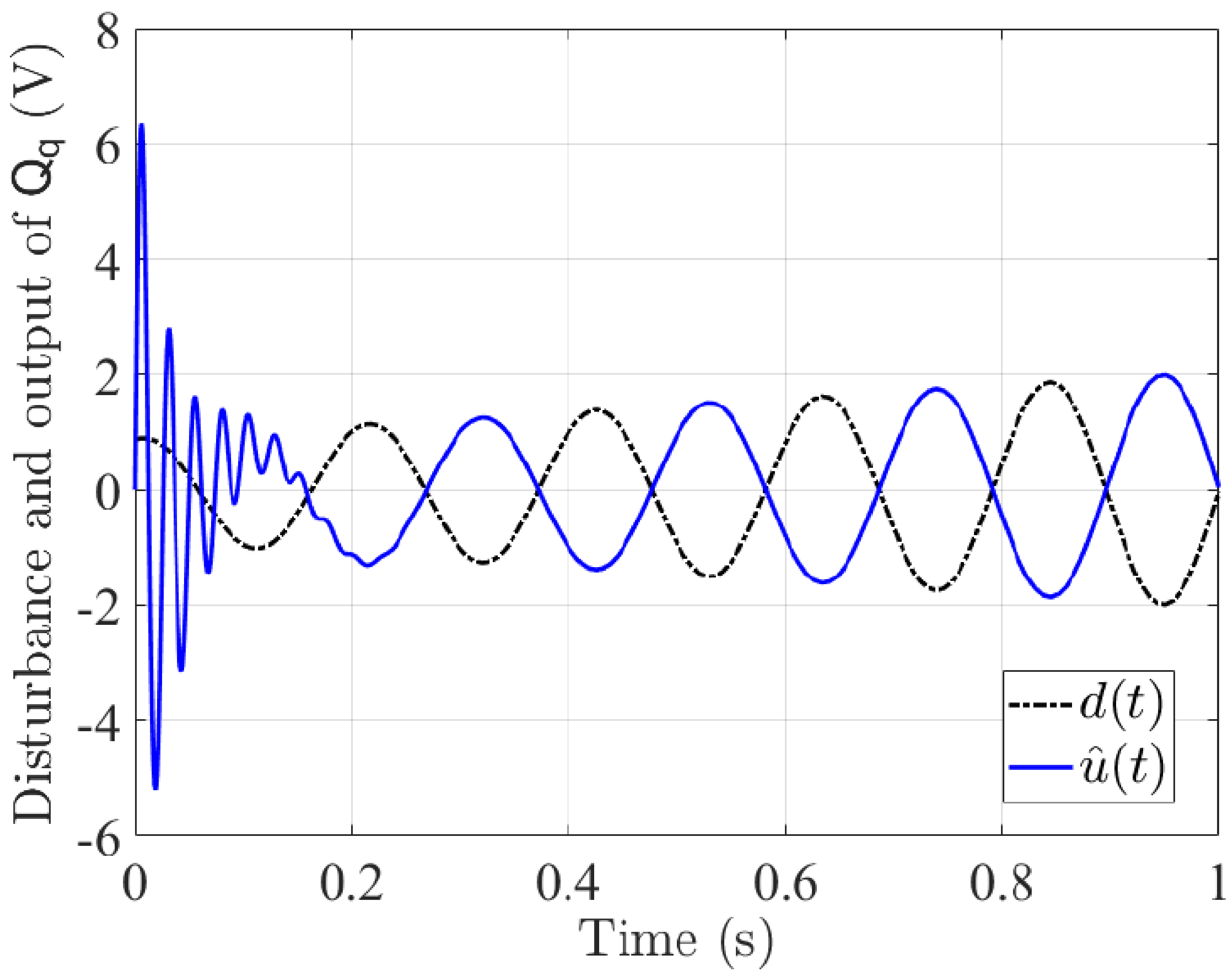
[6] Park, Shim, & Joo, "Recovering nominal tracking performance in an asymptotic sense for uncertain linear systems," *SIAM*, 2018.

Mechanical positioning system

Computer Simulation

- Initial conditions: Set to all zero except for $w(0) \in [-1, 1]^4$.
- Time-scale parameter: $\tau = 0.005$.

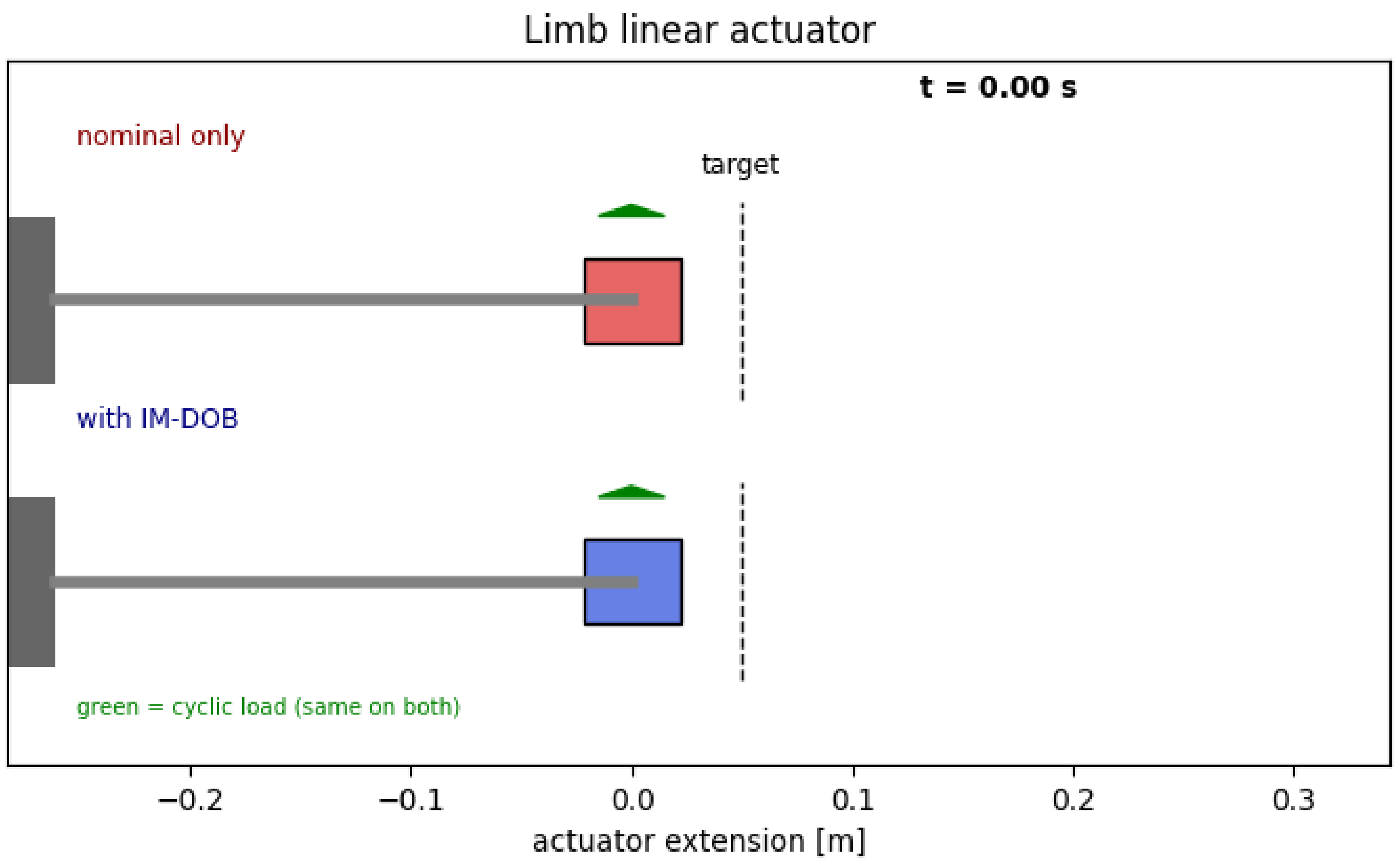




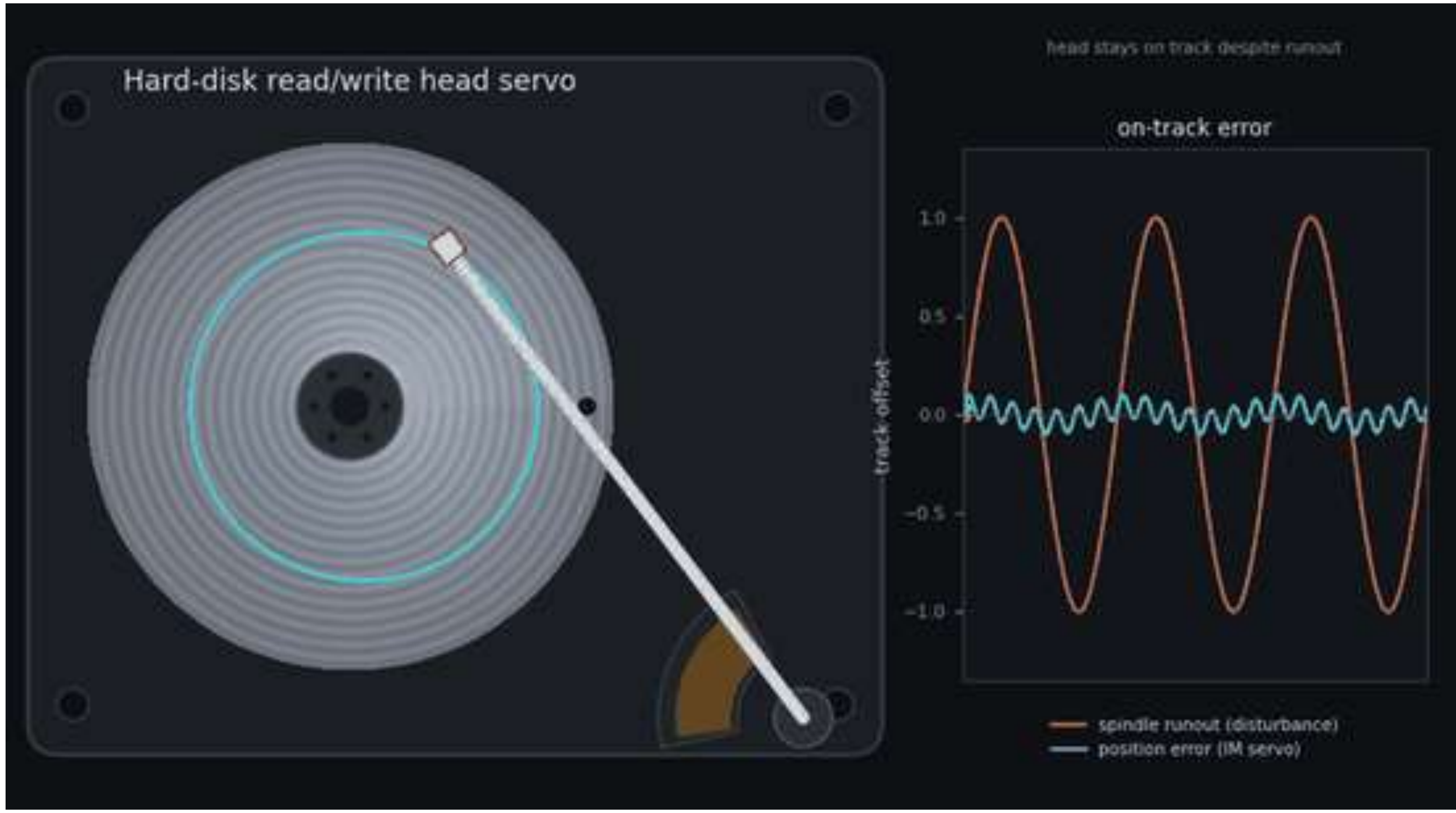
Note: $\hat{u}(t) = q_1(t) \to \Pi_1 w(t) = -Rw(t) = -d(t)$.

Real Systems that use disturbance observers

A linear actuator holds position against a **periodic load disturbance** (e.g. a gait-frequency cyclic force).



The read/write head must stay on a data track only tens of nanometres wide.



Conclusion

- We have designed an internal-model-based DOB for a class of linear systems, and shown the asymptotic stability of the closed-loop system.
- The matrix identity describing the internal model principle has played a crucial role.

$$\Pi S = (A_\mu - aC_\mu)\,\Pi, \qquad -R = C_\mu \Pi.$$

- The existing internal-model-based design of DOBs handles the disturbance by incorporating it into the quasi-steady-states, whereas our method explicitly takes it into account in the coordinate changes using the developed matrix identity. By this, we have

$$\lim_{t\to\infty} \|q(t) - \Pi w(t)\| = 0$$

and the stability analysis has been simplified compared to the existing works.

Thank you!